NATIONAL ADVISORY COMMITTEE
FOR AERONAUTICS

TECHNICAL NOTE 3233

A REVIEW OF PLANING THEORY AND EXPERIMENT WITH A

THEORETICAL STUDY OF PURE-PLANING LIFT OF

RECTANGULAR FLAT PLATES

By Charles L. Shuford, Jr.

Langley Aeronautical Laboratory
Langley Field, Va.

NACA

Washington

August 1954

K

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

TECHNICAL NOTE 3233

A REVIEW OF PLANING THEORY AND EXPERIMENT WITH A

THEORETICAL STUDY OF PURE-PLANING LIFT OF

RECTANGULAR FLAT PLATES

By Charles L. Shuford, Jr.

SUMMARY

A summary is given of the background and present status of the pure-planing flat-plate lift theories. The fundamental assumptions and the applicability to actual calculations of the planing lift force are reviewed.

A proposed theory based on the consideration of linear lifting-line theory less the suction component of lift plus crossflow effects is presented. A comparison of this theory with existing planing formulas and experimental data is made. The agreement between the results calculated by the proposed theory and the experimental data is satisfactory for engineering calculations of pure-planing rectangular-flat-plate lift and center of pressure.

INTRODUCTION

Recent developments in water-based aircraft have resulted in configurations utilizing planing surfaces operating in ranges of trim, length-beam ratio, and Froude number beyond those for which most of the available planing theories were correlated with experimental data. In order to determine whether available planing theories are adequate in estimating the planing lift in these extended ranges, a review of these theories (refs. 1 to 13) and a correlation with existing data, including recent and unpublished data, were made and are presented herein. For purposes of expediency and simplification, this work is limited to the case of the rectangular flat plate in pure planing, that is, where buoyancy can be considered as negligible.

In addition to this review and correlation, an additional theory for the lift and center of pressure of a rectangular flat plate was developed and correlated with the pure-planing data. The proposed theory distinguishes between linear and nonlinear components of lift and is divided

into three parts: first, a reasonably accurate approximation to the linear components of lift; second, an estimation of the aerodynamic leading-edge-suction component of lift contained in the linear term; and third, a method for calculating the crossflow effects.

SYMBOLS

A aspect ratio, b^2/S

b beam of planing surface, ft

C_L lift coefficient, $\dfrac{L}{\dfrac{\rho}{2}V^2 S}$

C_{L_B} lift coefficient due to buoyancy, $\dfrac{L_B}{\dfrac{\rho}{2}V^2 S}$

C_{L_b} lift coefficient based on square of beam, $\dfrac{\Delta}{\dfrac{\rho}{2}V^2 b^2}$

C_{L_S} lift coefficient based on principal wetted area,

$$\dfrac{\Delta}{\dfrac{\rho}{2}V^2 S} = \dfrac{C_{L_b}}{l_m/b}$$

C_N normal-force coefficient, $\dfrac{N}{\dfrac{\rho}{2}V^2 S}$

C_V speed coefficient or Froude number, $V/\sqrt{gb}$

g acceleration due to gravity, 32.2 ft/sec^2

L lift of planing surface, lb

L_B lift due to buoyancy, lb

l wetted length of planing surface or chord of airfoil, ft

$$l^* = \frac{l}{b/2}$$

l_m mean wetted length, ft

l_p center-of-pressure location (measured forward of trailing edge), ft

$\left(\dfrac{l_p}{l_m}\right)_{calc}$ nondimensional center-of-pressure location

m_0 section lift-curve slope per radian

N normal force, lb

q free-stream dynamic pressure, $\frac{1}{2}\rho V^2$, lb/sq ft

S principal wetted area (bounded by trailing edge, chines, and heavy spray line), sq ft

V horizontal velocity, fps

w induced vertical velocity

w_y induced vertical velocity at distance y from center line of airfoil

y distance from center line of airfoil to point where value of downwash is desired

$$y^* = \frac{y}{b/2}$$

Γ circulation or strength of vortex, $\dfrac{V l C_L}{2}$

γ nondimensional loading parameter, $C_L l^*$

Δ vertical load, lb

η distance from center line of airfoil to vortex

$$\eta^* = \frac{\eta}{b/2}$$

$$\theta = \cos^{-1}\eta*$$

$$\theta_1 = \cos^{-1}y*$$

ρ mass density of water, slugs/cu ft

τ trim (angle between planing bottom and horizontal), radians
                unless otherwise stated

τ_i induced angle of trim, w/V

τ_{i_y} induced angle of trim at distance y from center line of
                airfoil, w_y/V

φ Pabst's aspect-ratio correction factor based on the ratio
                of wetted length to mean beam

REVIEW OF EXISTING PLANING-LIFT THEORY

Wagner (ref. 1) considered the planing-force problem theoretically; however, his work is valuable mainly for the basic concepts presented in the application of the methods of airfoil theory to the planing problem. Wagner's work consists of studies of the flow processes and solutions for the force on an idealized two-dimensional planing surface; therefore, his work is not directly applicable for calculating the lift on a finite-aspect-ratio planing surface.

In planing theories such as that of Mayo (ref. 2) developed from virtual-mass considerations based on transverse flow, the assumption is made that the planing force can be calculated from the rate at which momentum is imparted to the downwash; however, the effect of aspect ratio is approximated by the Pabst empirical aspect-ratio correction factor (ref. 14).

In reference 3 Sokolov presented a combined theoretical and experimental solution of the planing problem. The theoretical formulas, which were developed for the two-dimensional case, were derived by using Bernoulli's equation and disturbance velocities. A finite-aspect-ratio planing-lift formula was developed by using Sottorf's experimental results (ref. 15) to determine empirically the value of the factor ϵ which is the ratio of the change in velocity along the planing surface to the velocity of the free stream. The planing formula gives the lift forces in three components: the hydrostatic, the one due to circulation, and the one due to form. The solution given by Sokolov for pure-planing flat-plate lift is

$$C_{L_S} = \epsilon(2 - \epsilon)\cos \tau \tag{1}$$

where curves for ϵ are given in reference 3.

Sokolov gives a qualitative picture of the planing problem and determines the nature of the forces involved. The concepts presented, however, have not been used in the development of subsequent planing formulas, which have been empirical or follow the work of Wagner.

Perring and Johnston (ref. 4) presented the empirical relationship

$$C_{L_S} = CA^n\tau \tag{2}$$

and by analyzing Sottorf's data (ref. 15) found the following formula to apply:

$$C_{L_S} = 0.90A^{0.42}\tau \tag{3}$$

In reference 5, Sottorf proposed the formula

$$C_{L_S} = 0.845A^{0.5}\tau \tag{4}$$

An equation that has a form similar to airfoil lifting-line theory was presented by Perelmuter (ref. 6). The equation is

$$C_{L_S} = \frac{2A\tau}{1 + A} \tag{5}$$

Sedov (ref. 7) gives an equation based on the data of Sottorf (ref. 15) and Sambraus (ref. 16) which has the form

$$C_{L_S} = \frac{0.7\pi A \tau}{A + 1.4} \tag{6}$$

An equation that contains a linear and nonlinear term was presented by Siler in reference 8. The linear term was obtained by assuming a form similar to airfoil lifting-line theory; however, the aspect-ratio factor was altered to give a deflected mass one-half that predicted by Jones (ref. 17) for a zero-aspect-ratio wing. The nonlinear term was obtained by a consideration of the transverse component of the flow (see ref. 18). The equation can be written in the form

$$C_{L_S} = \frac{\pi A \sin \tau \cos \tau}{A + 4} + 0.88 \sin^2\tau \cos \tau \tag{7}$$

In reference 9, Korvin-Kroukovsky, Savitsky, and Lehman proposed an equation derived primarily on the basis of the data of Sottorf (ref. 15) and Sambraus (ref. 16). The formula can be written as

$$C_{L_S} = 0.012A^{0.5}(57.3\tau)^{1.1} \tag{8}$$

In reference 10, Korvin-Kroukovsky presented an equation that consisted of linear and nonlinear components. The linear term was obtained by a consideration of the downwash and the analytical solution for the potential flow about a planing surface developed by Wagner (ref. 1) and presented in detail by Pierson and Leshnover (ref. 19). The nonlinear term was obtained by a consideration of the transverse component of the flow. The equation has the form

$$C_{L_S} = \frac{\pi A \tau}{A + 2} + 0.88\tau^2$$

However, this equation was empirically corrected to get better agreement with experimental data, so that approximately

$$C_{L_S} = \frac{0.73\pi A \tau}{A + 2} + 0.88\tau^2 \tag{9}$$

Locke (ref. 11) assumed that the lift characteristics of low-aspect-ratio surfaces can be represented by a simple power function of the form

$$C_{L_S} = DK\tau^n$$

where K and n depend only on aspect ratio and D is primarily a function of the operating conditions. For the case of the flat-plate planing surface Locke gives the equation

$$C_{L_S} = 0.5K\tau^n \qquad (10)$$

where curves for K and n are given in reference 11.

In reference 12 Perry assumed an equation for the ratio of planing lift to aerodynamic lift which converged to limits obtained by applying airfoil methods to the planing surface. The equation has the form

$$C_{L_S} = MC_{L_{airfoil}} \qquad (11)$$

where M represents the assumed equation for the ratio of planing lift to aerodynamic lift given by

$$M = \frac{\mu \dfrac{A}{2} + \dfrac{2}{\pi + 4} \sin \tau}{\dfrac{A}{2} + \dfrac{2}{\pi} \sin \tau}$$

and

$$\mu = \frac{\cos \tau}{1 + \cos \tau - (1 - \cos \tau)\log_e\left(\dfrac{1 - \cos \tau}{2 \cos \tau}\right) + \pi \sin \tau}$$

Curves for M and μ are given in reference 12.

The limit of M for zero aspect ratio is 0.88 and for infinite aspect ratio is the value μ. The value of M for zero aspect ratio is a result given by Bollay (ref. 18). The value of M for infinite aspect

ratio was obtained by a consideration of the analytical solution for the potential flow about a planing surface developed by Wagner (ref. 1) and presented in detail by Pierson and Leshnover (ref. 19).

An equation having a linear term with a form analogous to airfoil lifting-surface theory was proposed by P. R. Crewe of Saunders-Roe Ltd. (British) in correspondence between himself and the Langley Laboratory. This equation, based on the data of Kapryan and Weinstein (ref. 20), is

$$C_{L_S} = \sin \tau \cos \tau \left[\frac{8}{\pi} \frac{1}{1 + \sqrt{1 + \left(\frac{2}{A}\right)^2}} + 2 \sin \tau - B \sin^2 \tau \right] \qquad (12)$$

where

$$B = 2.67 \qquad\qquad (A < 2.0)$$

$$B = 3.0 \qquad\qquad (A > 2.0)$$

Schnitzer (ref. 13) presented an equation derived from a consideration of two-dimensional deflected mass, modified for three-dimensional flow by the Pabst empirical aspect-ratio correction factor (ref. 14). The equation can be written in the form

$$C_{L_S} = \varphi \left(\frac{\pi^3 A}{16} \sin \tau \cos \tau + 0.88 \sin^2 \tau \right) \qquad (13)$$

PROPOSED THEORY

An examination of experimental data indicates a pronounced nonlinear relationship between the planing lift coefficients and the angle of attack; therefore, linear theory would not provide adequate approximations to the planing lift. The determination of linear and nonlinear components of lift is the approach generally used in low-aspect-ratio airfoil theory. The present approach is based on the consideration of linear lifting-line theory less the suction component of lift plus viscous crossflow effects.

Lift

Linear term.- The linear term is determined from a consideration of lifting-line airfoil theory. Since the heavy spray line (leading edge) of a planing surface is approximately elliptic, the airfoil theory is presented for an elliptic surface with elliptic loading and then modified for the planing case.

By use of the Prandtl airfoil theory, the airfoil lift coefficient is

$$C_L = m_0(\tau - \tau_i) \tag{14}$$

where m_0 is the slope of the section lift curve.

If a sheet of trailing vortices located at 0.75 of the chord measured from the trailing edge and extending to infinity behind the airfoil is assumed, then from vortex theory the induced angle of attack is

$$\tau_{i_y} = \frac{w_y}{V} = \frac{1}{4\pi V} \int_{-b/2}^{b/2} \frac{-d\Gamma}{d\eta} \frac{d\eta}{(\eta - y)} \tag{15}$$

where η is the distance from the center line of the airfoil to the vortex and y is the distance from the center line of the airfoil to the point where the value of downwash is desired.

Now let

$$y^* = \frac{y}{b/2} = \cos \theta_1 \tag{16}$$

$$\eta^* = \frac{\eta}{b/2} = \cos \theta \tag{17}$$

$$l^* = \frac{l}{b/2} \tag{18}$$

and

$$\gamma = \frac{4\Gamma}{bV} \tag{19}$$

where

$$\gamma = C_L l^* = m_0 l^* (\tau - \tau_i) \tag{20}$$

From equation (15),

$$\tau_{i_y} = \frac{1}{8\pi} \int_0^\pi \frac{d\gamma}{d\theta} \frac{d\theta}{\cos \theta - \cos \theta_1}$$

Now let

$$\gamma = \sum_{n=1}^\infty a_n \sin n\theta \tag{21}$$

then

$$\tau_{i_y} = \frac{1}{8\pi} \int_0^\pi \frac{\sum_{n=1}^\infty n a_n \cos n\theta \, d\theta}{\cos \theta - \cos \theta_1} \tag{22}$$

The solution of this equation can be obtained by means of a recurrence formula and the solution of a linear finite-difference equation (ref. 21); thus,

$$\tau_{i_y} = \sum_{n=1}^\infty \frac{n a_n}{8} \frac{\sin n\theta_1}{\sin \theta_1} \tag{23}$$

Since this equation is valid for any value of θ_1, the subscripts on τ_i and θ can be dropped and equation (20) becomes

$$\tau = \sum_{n=1}^\infty \left(\frac{1}{m_0 l^*} + \frac{n}{8 \sin \theta} \right) a_n \sin n\theta \tag{24}$$

For an elliptic airfoil,

$$l^* = \frac{8}{\pi A} \sin \theta \qquad (25)$$

and

$$\tau \sin \theta = \sum_{n=1}^{\infty} \frac{1}{8}\left[\left(\frac{\pi A}{m_0} + n\right)\sin n\theta\right] a_n$$

Let

$$\tau \sin \theta = \sum_{n=1}^{\infty} b_n \sin n\theta \qquad (26)$$

then

$$a_n = \frac{8b_n}{\dfrac{\pi A}{m_0} + n} \qquad (27)$$

where

$$b_n = \frac{2}{\pi}\int_0^\pi (\tau \sin \theta)\sin n\theta \, d\theta \qquad (28)$$

Let τ be constant along the wing; then,

$$b_1 = \tau \qquad b_2 = b_3 = b_{n\neq1} = 0$$

$$a_n = a_1 \qquad a_2 = a_3 = a_{n\neq1} = 0$$

The lift coefficient is given by

$$C_L = \int_{-b/2}^{b/2} \frac{\rho V \Gamma}{qS} \, d\eta \qquad (29)$$

By use of equations (17), (19), and (21), equation (29) becomes

$$C_L = \frac{A}{4} \int_0^\pi \gamma \sin \theta \, d\theta = \frac{\pi}{8} A a_n \tag{30}$$

Therefore, from equations (27) and (28),

$$C_L = \frac{\pi A \tau}{\dfrac{\pi A}{m_0} + 1} \tag{31}$$

which is the equation for the lift on an airfoil.

For a low-aspect-ratio planing surface having flow only on one side, the lift coefficient is assumed to be one-half the value given by equation (31) for a flat-plate airfoil, and m_0, the lift-curve slope for the two-dimensional planing surface, is assumed to be one-half the value given for a flat-plate airfoil; thus,

$$C_L = \frac{0.5 \pi A \tau}{A + 1} \tag{32}$$

which gives the linear component of lift on a pure-planing flat plate.

Suction component of lift.- An airfoil has a suction component of lift due to the large negative pressures at the leading edge of the airfoil; however, for a planing surface this suction component of lift does not appear. Therefore, the lift obtained from the linear term (eq. (32)) is less by an amount equal to the suction component of lift given by

$$C_L = C_{L_{linear}} \sin^2 \tau \tag{33}$$

which is the value indicated by Wagner in reference 22.

The linear term (eq. (32)) less the suction component of lift is

$$C_L = \frac{0.5 \pi A \tau}{1 + A} \left(1 - \sin^2 \tau \right) \tag{34}$$

Crossflow term.- For a simple theoretical consideration of the non-
linear term, the velocity component perpendicular to the chord is assumed
to be of the magnitude $V \sin \tau$. The drag coefficient for a planing sur-
face of infinite aspect ratio is assumed to be 1.0, which is one-half the
value given for a two-dimensional flat-plate airfoil. For the planing
surface the flow is projected into components perpendicular and parallel
to the planing-surface chord line, and the drag force associated with the
flow perpendicular to the chord is calculated. Therefore, the normal
force is

$$N = 1.0 \, \frac{\rho}{2} \, S(V \sin \tau)^2$$

and

$$C_N = \sin^2\tau$$

or

$$C_L = \sin^2\tau \cos \tau \qquad (35)$$

which is a lift due to crossflow effects and is proportional to $\sin^2\tau$,
which is the concept presented for airfoils by Betz in reference 23.

Total lift.- The total lift on a pure-planing rectangular flat plate
can be obtained by adding equations (34) and (35) and is

$$C_{L_S} = \frac{0.5\pi A\tau}{1 + A}\left(1 - \sin^2\tau\right) + \sin^2\tau \cos \tau \qquad (36)$$

which represents the linear term less the suction component of lift plus
the crossflow term. The magnitude of the crossflow effects, total lift,
and total lift excluding suction effects is shown in figure 1.

Comparison of proposed and previous planing formulas.- A comparison
of the proposed theory with previous planing formulas for constant length-
beam ratios is given in figure 2. In figure 2(a) the proposed theory is
compared with the planing formulas as presented by Sokolov (eq. (1)),
Perring and Johnston (eq. (3)), Sottorf (eq. (4)), Perelmuter (eq. (5)),
and Sedov (eq. (6)). In figure 2(b) the proposed theory is compared with
the planing formulas presented by Siler (eq. (7)), Korvin-Kroukovsky
(eq. (9)), and Schnitzer (eq. (13)). In figure 2(c) the proposed theory

is compared with the planing formulas presented by Korvin-Kroukovsky, Savitsky, and Lehman (eq. (8)), Locke (eq. (10)), and Crewe (eq. (12)).

The values given by the formula presented by Perry (ref. 12) were not plotted since the results depended on the airfoil data used. Perry showed that by using Winter's airfoil data (ref. 24) his formula approximated the results given by the formula presented by Korvin-Kroukovsky, Savitsky, and Lehman (ref. 9) for trims up to 12° and length-beam ratios below approximately 1.0.

Center of Pressure

The center of pressure on a planing surface of small aspect ratio may be considered to have two components, the component due to the linear-lift term less the suction effects and the component due to the crossflow term. The center of pressure for the lift due to the linear component of lift less the suction effects (eq. (34)) is assumed to be located at 0.75 of the mean wetted length from the trailing edge of the planing surface. The center of pressure for the lift due to the crossflow term (eq. (35)) is assumed to be located at the center of the mean wetted length; therefore,

$$\left(\frac{l_p}{l_m}\right)_{calc} = \frac{0.75\left(C_{L_{total}} - C_{L_{crossflow}}\right) + 0.5C_{L_{crossflow}}}{C_{L_{total}}} \qquad (37)$$

which is a formula analogous to that used in airfoil theory. The components of lift are determined from equations (35) and (36).

COMPARISON OF THEORY AND EXPERIMENT

Buoyancy

The experimental data were considered as pure planing if the lift coefficient due to buoyancy, calculated from the wedge-shaped volumetric displacement of the planing surface below the level water surface and given by

$$C_{L_B} = \frac{l_m}{b} \frac{1}{2C_V{}^2} \sin 2\tau \qquad (38)$$

did not exceed a given value. The allowable lift coefficient due to
buoyancy, as determined from equation (38), was arbitrarily selected as
0.01 at a trim of 16°. The allowable lift coefficient due to buoyancy for
other trims was determined by drawing a straight line from zero trim and
zero lift coefficient due to buoyancy through the value 0.01 at a trim
of 16°. The permissible lift coefficient due to buoyancy for the data
selected by this method at a trim of 2° varied from 16 percent at a length-
beam ratio of 8 to 3.3 percent at a length-beam ratio of one-half. These
values decreased with increasing trim so that at 30° they would vary from
6.5 percent at a length-beam ratio of 8 to 3.1 percent at a length-beam
ratio of one-half. Analysis of unpublished Langley tank no. 2 data showed
that subtraction of the lift coefficient due to buoyancy computed from
equation (38) caused the data to approximately coincide (or collapse)
for different speeds at low trims (4°); however, at the higher trims
(16°) only approximately one-half of this value was needed to collapse
the data.

Lift

A comparison of the proposed theory with the experimental data of
Weinstein and Kapryan (ref. 25), unpublished NACA data, data of Shoemaker
(ref. 26), data of Locke (ref. 27), data of Sambraus (ref. 16), and data
of Sottorf (ref. 15) is presented in figures 3 to 10. Only the experi-
mental data indicated as pure planing by the method discussed in the
preceding section have been considered.

Figure 3 gives a comparison of the proposed theory with the data of
Weinstein and Kapryan (ref. 25). Figure 4 gives a comparison of the pro-
posed theory with unpublished Langley tank no. 2 data. The data of
Weinstein and Kapryan were obtained for a 4-inch-beam model tested at
various loads and speeds and the unpublished data were from a 2.5-inch-
beam model tested at a constant speed of 30 feet per second.

In figures 5 to 10 a comparison of experimental lift coefficients
given in references 25, 26, 27, 16, and 15 and unpublished NACA data
with the proposed formulas given by Crewe (eq. (12)), Locke (eq. (10)),
Korvin-Kroukovsky, Savitsky, and Lehman (eq. (8)), and the present paper
is presented. In general, the proposed theory gives an average of these
data. The formula presented by Crewe (eq. (12)) is in good agreement,
except at a trim of 30°, with the data of Weinstein and Kapryan (figs. 5(a)
and 5(b)), with the data of Sambraus (fig. 9), and with the data of Sottorf
(fig. 10). The formulas presented by Locke (eq. (10)) and Korvin-
Kroukovsky, Savitsky, and Lehman (eq. (8)) do not give so good a repre-
sentation of experimental data as the proposed theory or the formulas
presented by Crewe (eq. (12)). There are no experimental data at high
trims and large length-beam ratios to determine whether the planing for-
mulas give the correct variation of lift in this region. The agreement

between the proposed theory and experiment is apparently satisfactory for engineering calculations of pure-planing rectangular-flat-plate lift in the ranges where experimental data are available.

Center of Pressure

The variation of center-of-pressure ratio with mean wetted-length—beam ratio for the data of Weinstein and Kapryan (ref. 25) is shown in figure 11 and for the unpublished Langley tank no. 2 data in figure 12. The proposed theory is shown by the solid line in figures 11 and 12 where

$\dfrac{l_p}{b} = \left(\dfrac{l_p}{l_m}\right)_{calc} \dfrac{l_m}{b}$. The agreement between the curve for the proposed

theory and the experimental points appears to be satisfactory for engineering calculations of pure-planing rectangular-flat-plate center of pressure in the ranges where experimental data are available.

CONCLUDING REMARKS

The proposed theory appears to predict the pure-planing rectangular-flat-plate lift and center of pressure with engineering accuracy in the ranges where experimental data are available; however, at high trims and large wetted-length—beam ratios no data are available. The correlation of experimental data and theory in this report seems to establish firmly the utility of the $\sin^2\tau$ approach (where τ is trim) to the nonlinearity problem.

Langley Aeronautical Laboratory,
    National Advisory Committee for Aeronautics,
        Langley Field, Va., June 10, 1954.

REFERENCES

1. Wagner, Herbert: Phenomena Associated With Impacts and Sliding on
    Liquid Surfaces. S-302, British A.R.C., July 14, 1936.

2. Mayo, Wilbur L.: Analysis and Modification of Theory for Impact of
    Seaplanes on Water. NACA Rep. 810, 1945. (Supersedes NACA TN 1008.)

3. Sokolov, N. A.: Hydrodynamic Properties of Planing Surfaces and Flying
    Boats. NACA TM 1246, 1950.

4. Perring, W. G. A., and Johnston, L.: Hydrodynamic Forces and Moments
    on a Simple Planing Surface and on a Flying Boat Hull. R. & M.
    No. 1646, British A.R.C., 1935.

5. Sottorf, W.: Analysis of Experimental Investigations of the Planing
    Process on the Surface of Water. NACA TM 1061, 1944.

6. Perelmuter, A.: On the Determination of the Take-Off Characteristics
    of a Seaplane. NACA TM 863, 1938.

7. Sedov, L.: Scale Effect and Optimum Relations for Sea Surface Planing.
    NACA TM 1097, 1947.

8. Siler, William: Lift and Moment of Flat Rectangular Low Aspect Ratio
    Lifting Surfaces. Tech. Memo. No. 96, Exp. Towing Tank, Stevens Inst.
    Tech., 1949.

9. Korvin-Kroukovsky, B. V., Savitsky, Daniel, and Lehman, William F.:
    Wetted Area and Center of Pressure of Planing Surfaces. Preprint
    No. 244, S.M.F. Fund Paper, Inst. Aero. Sci. (Rep. No. 360, Project
    No. NR062-012, Office Naval Res., Exp. Towing Tank, Stevens Inst.
    Tech., Aug. 1949.)

10. Korvin-Kroukovsky, B. V.: Lift of Planing Surfaces. Jour. Aero. Sci.
    (Readers' Forum), vol. 17, no. 9, Sept. 1950, pp. 597-599.

11. Locke, F. W. S., Jr.: An Empirical Study of Low Aspect Ratio Lifting
    Surfaces With Particular Regard to Planing Craft. Jour. Aero. Sci.,
    vol. 16, no. 3, Mar. 1949, pp. 184-188.

12. Perry, Byrne: The Effect of Aspect Ratio on the Lift of Flat Planing
    Surfaces. Rep. No. E-24.5 (Contract N6 onr-24424, Project NR
    234-001), Hydrod. Lab., C.I.T., Sept. 1952.

13. Schnitzer, Emanuel: Theory and Procedure for Determining Loads and
    Motions in Chine-Immersed Hydrodynamic Impacts of Prismatic Bodies.
    NACA Rep. 1152, 1953. (Supersedes NACA TN 2813).

14. Pabst, Wilhelm: Landing Impact of Seaplanes.   NACA TM. 624, 1931.

15. Sottorf, W.:   Experiments With Planing Surfaces.   NACA TM 661, 1932.

16. Sambraus, A.:   Planing-Surface Tests at Large Froude Numbers - Airfoil
    Comparison.   NACA TM 848, 1938.

17. Jones, Robert T.:   Properties of Low-Aspect-Ratio Pointed Wings at
    Speeds Below and Above the Speed of Sound.   NACA Rep. 835, 1946.
    (Supersedes NACA TN 1032.)

18. Bollay, William: A Contribution to the Theory of Planing Surfaces.
    Proc. Fifth Int. Cong. Appl. Mech. (Cambridge, Mass., 1938), John
    Wiley & Sons, Inc., 1939, pp. 474-477.

19. Pierson, John D., and Leshnover, Samuel: An Analysis of the Fluid
    Flow in the Spray Root and Wake Regions of Flat Planing Surfaces.
    Preprint No. 166, S.M.F. Fund Paper, Inst. Aero. Sci. (Rep.
    No. 335, Project No. NR 062-012, Office of Naval Res., Exp. Towing
    Tank, Stevens Inst. Tech.), Oct. 1948.

20. Kapryan, Walter J., and Weinstein, Irving: The Planing Character-
    istics of a Surface Having a Basic Angle of Dead Rise of 20° and
    Horizontal Chine Flare.   NACA TN 2804, 1952.

21. Glauert, H.:   The Elements of Aerofoil and Airscrew Theory.   Second
    ed., Cambridge Univ. Press, 1947 (reprinted 1948), p. 92.

22. Wagner, Herbert: Planing of Watercraft.   NACA TM 1139, 1948.

23. Betz, A.:   Applied Airfoil Theory.   Airfoils or Wings of Finite Span.
    Vol. IV of Aerodynamic Theory, div. J, ch. III, sec. 7, W. F. Durand,
    ed., Julius Springer (Berlin), 1935 (reprinted by Durand Reprinting
    Committee, 1943), p. 69.

24. Winter, H.:   Flow Phenomena on Plates and Airfoils of Short Span.
    NACA TM 798, 1936.

25. Weinstein, Irving, and Kapryan, Walter J.:   The High-Speed Planing
    Characteristics of a Rectangular Flat Plate Over a Wide Range of
    Trim and Wetted Length.   NACA TN 2981, 1953.

26. Shoemaker, James M.:   Tank Tests of Flat and V-Bottom Planing Sur-
    faces.   NACA TN 509, 1934.

27. Locke, F. W. S., Jr.:   Tests of a Flat Bottom Planing Surface To
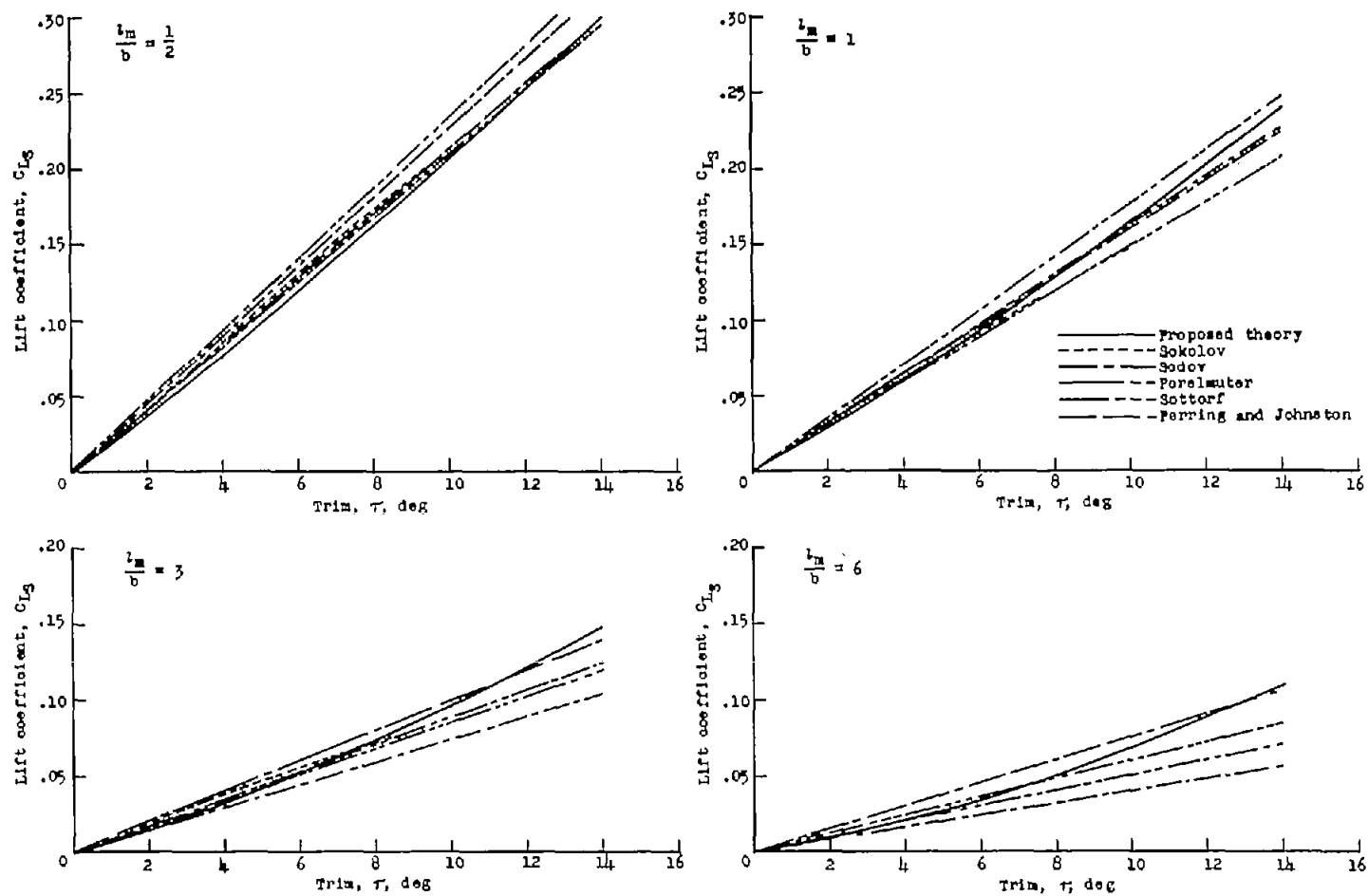   Determine the Inception of Planing.   NAVAER DR Rep. 1096, Bur.
    Aero., Dec. 1948.
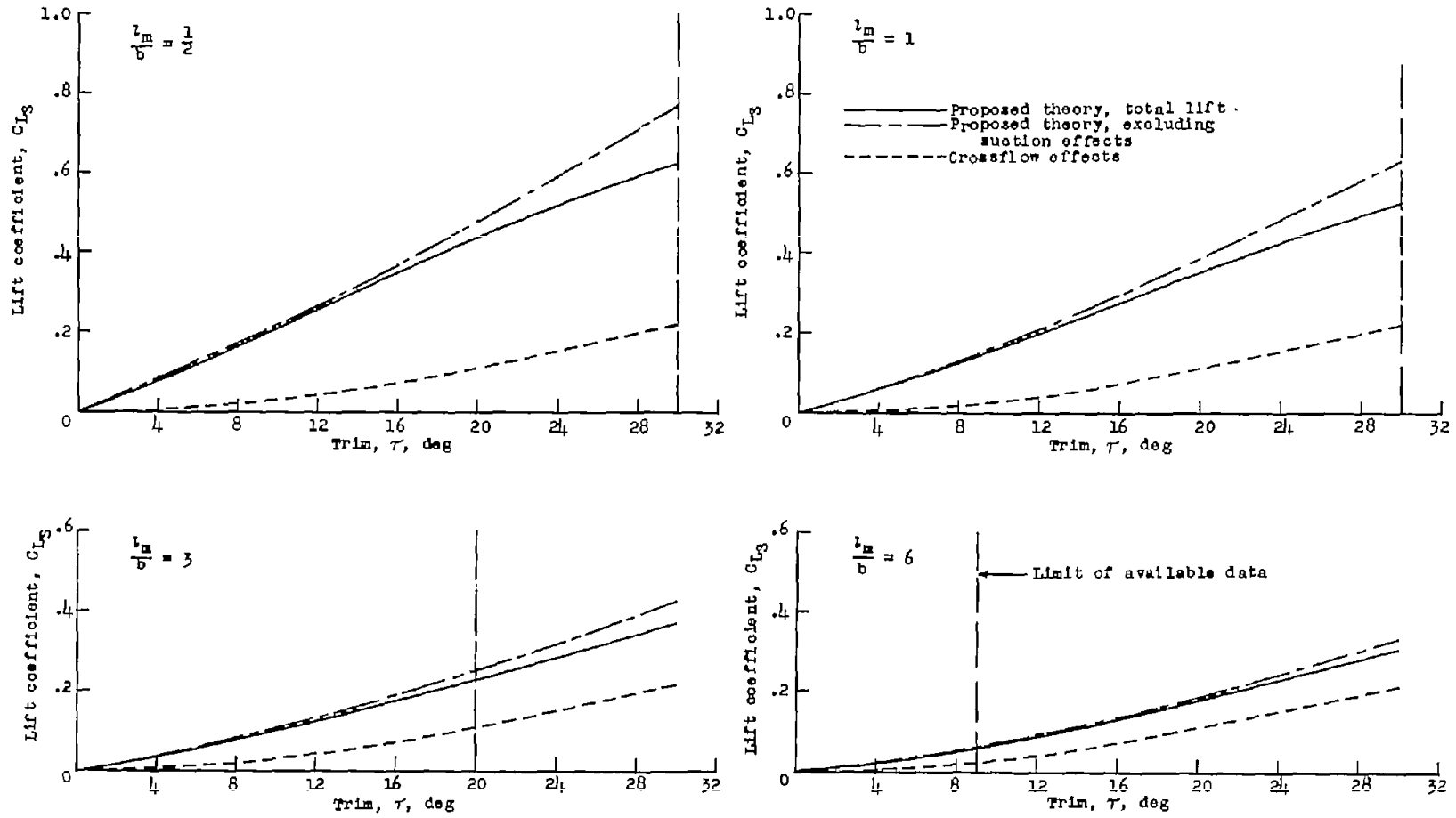
Figure 1.- Relative magnitude of components of proposed theory.

$\frac{l_m}{b} = \frac{1}{2}$

$\frac{l_m}{b} = 1$

Proposed theory
Sokolov
Sodov
Poralmuter
Sottorf
Perring and Johnston
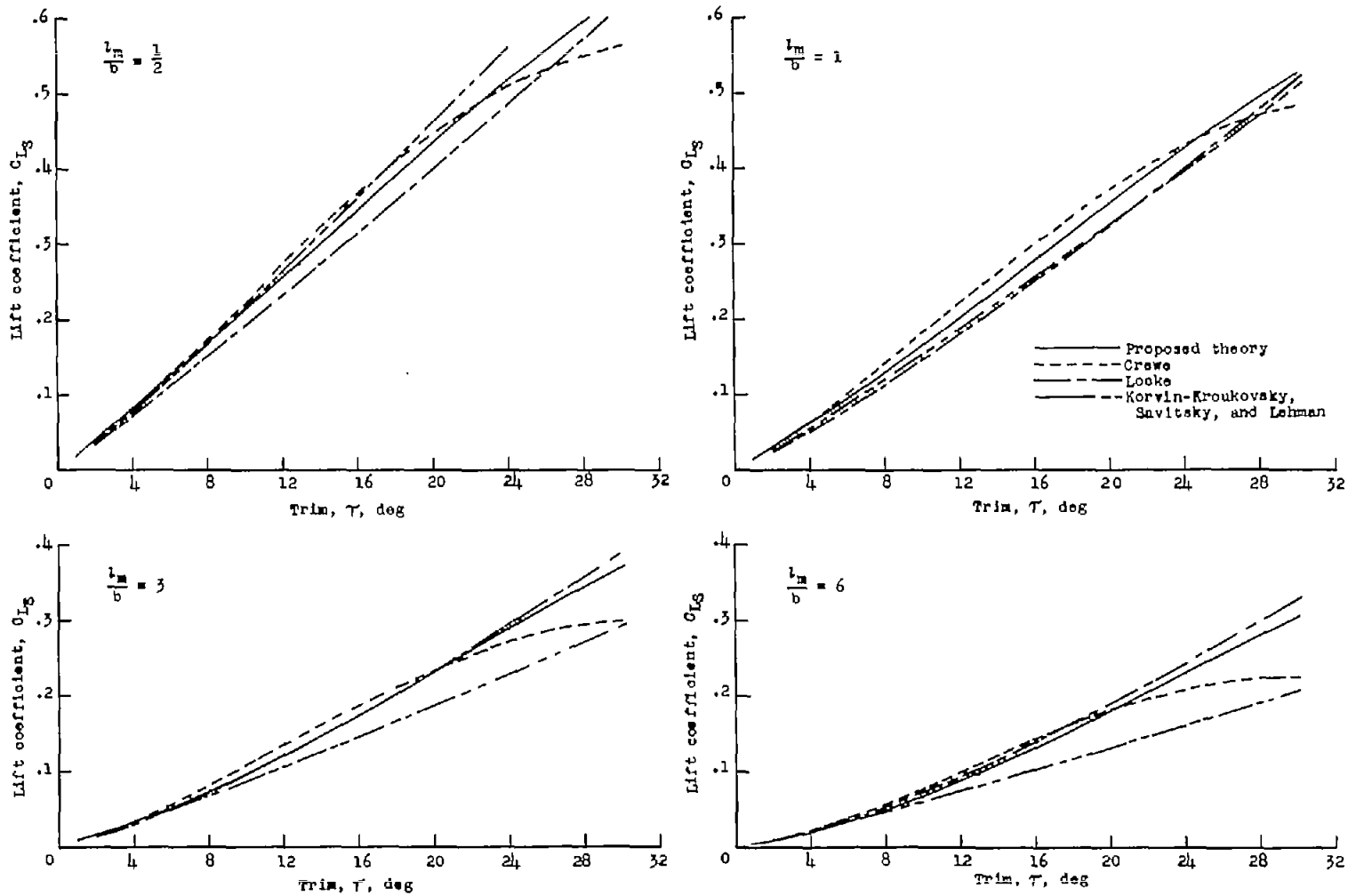
$\frac{l_m}{b} = 3$

$\frac{l_m}{b} = 6$

(a) Proposed theory and references 3 to 7.

Figure 2.- Variation of lift coefficient with angle of attack for rectangular-flat-plate lift formulas.
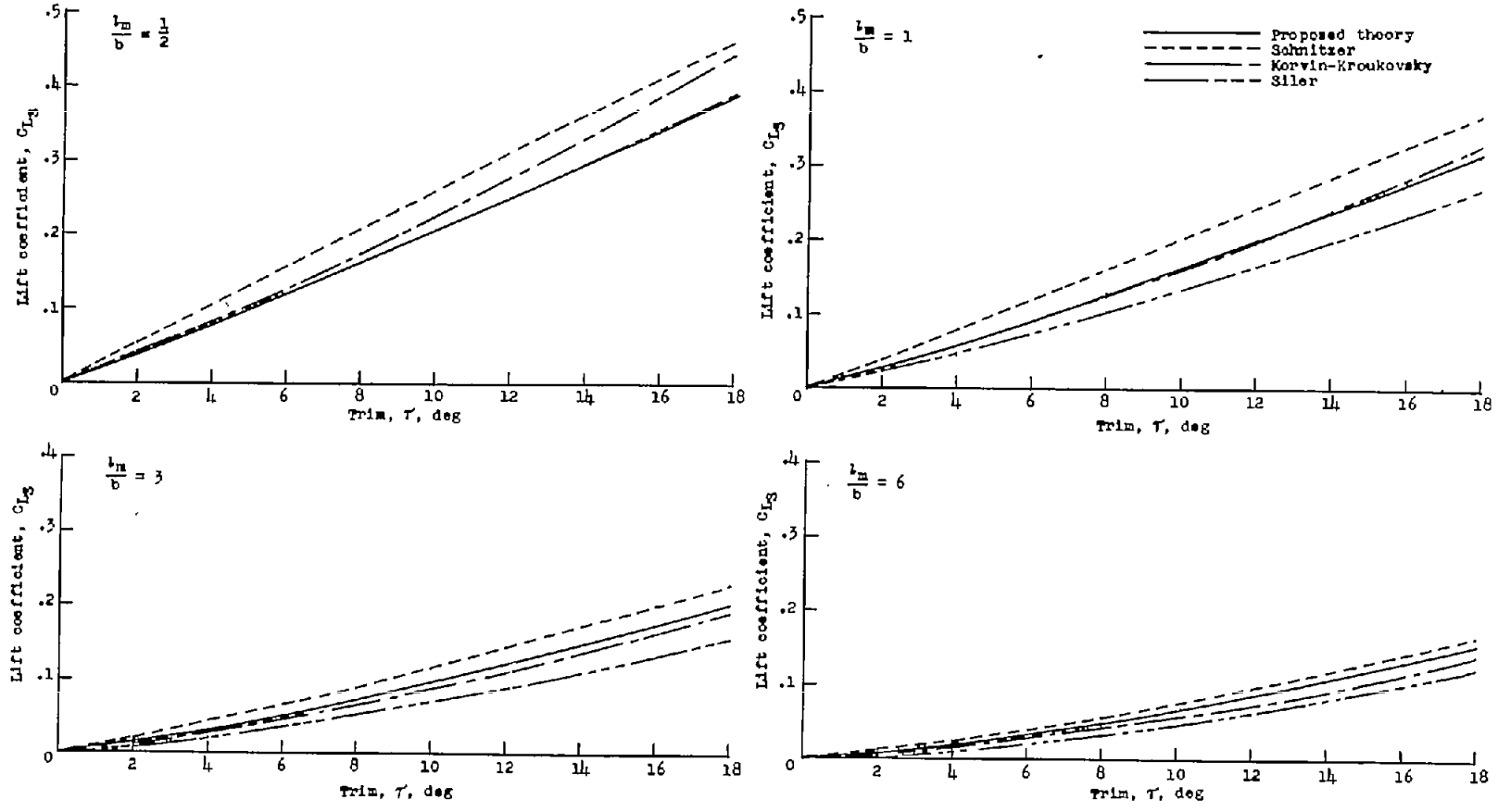
(b) Proposed theory and references 8, 10, and 13.

Figure 2.- Continued.

(c) Proposed theory, references 9 and 11, and Crewe's equation (eq. (12)).

Figure 2.- Concluded.
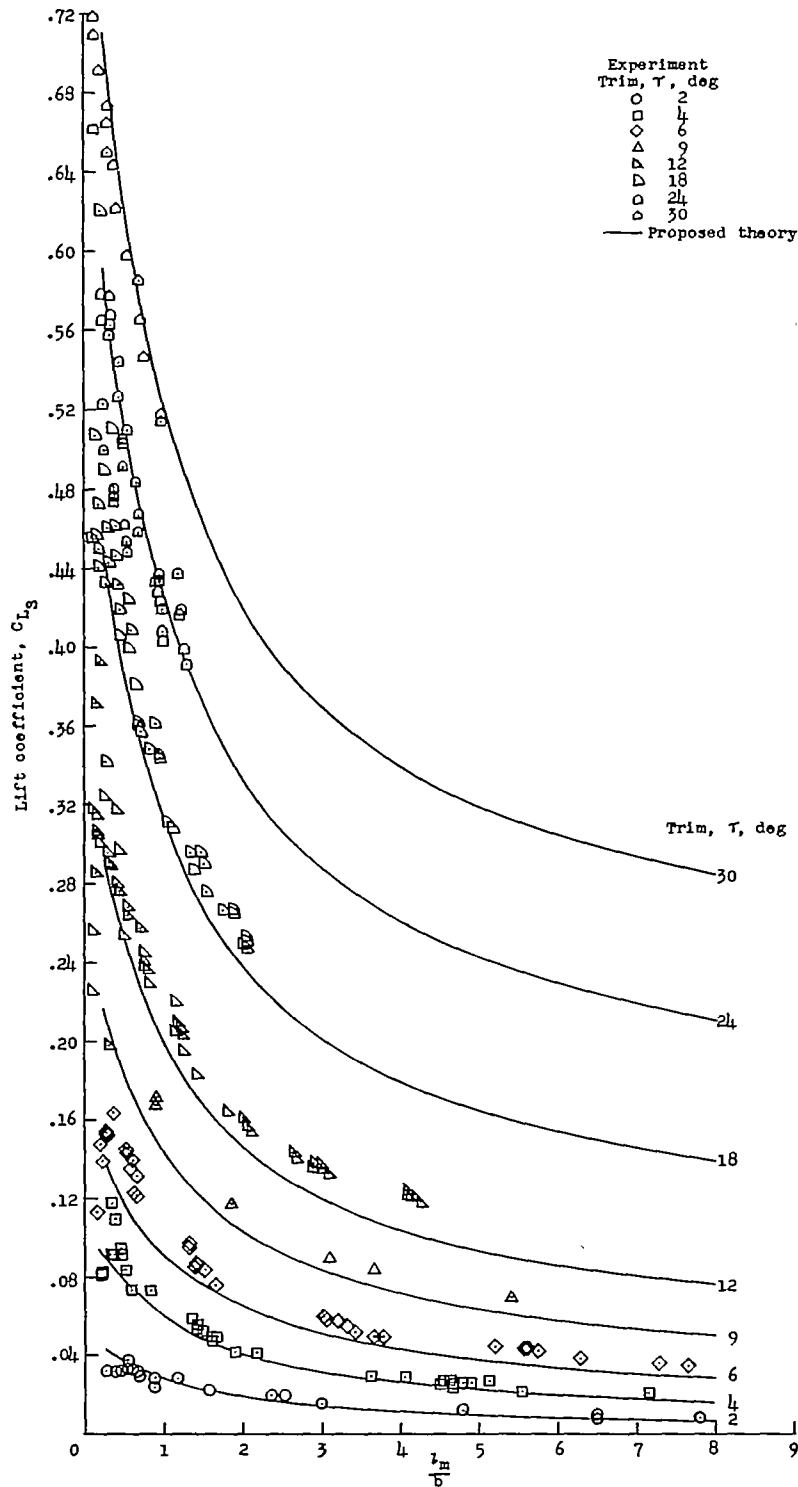
Figure 3.- Comparison of proposed theory with experimental lift coeffi-
cients for the 4-inch-beam rectangular-flat-plate planing-surface data
of Weinstein and Kapryan (ref. 25).
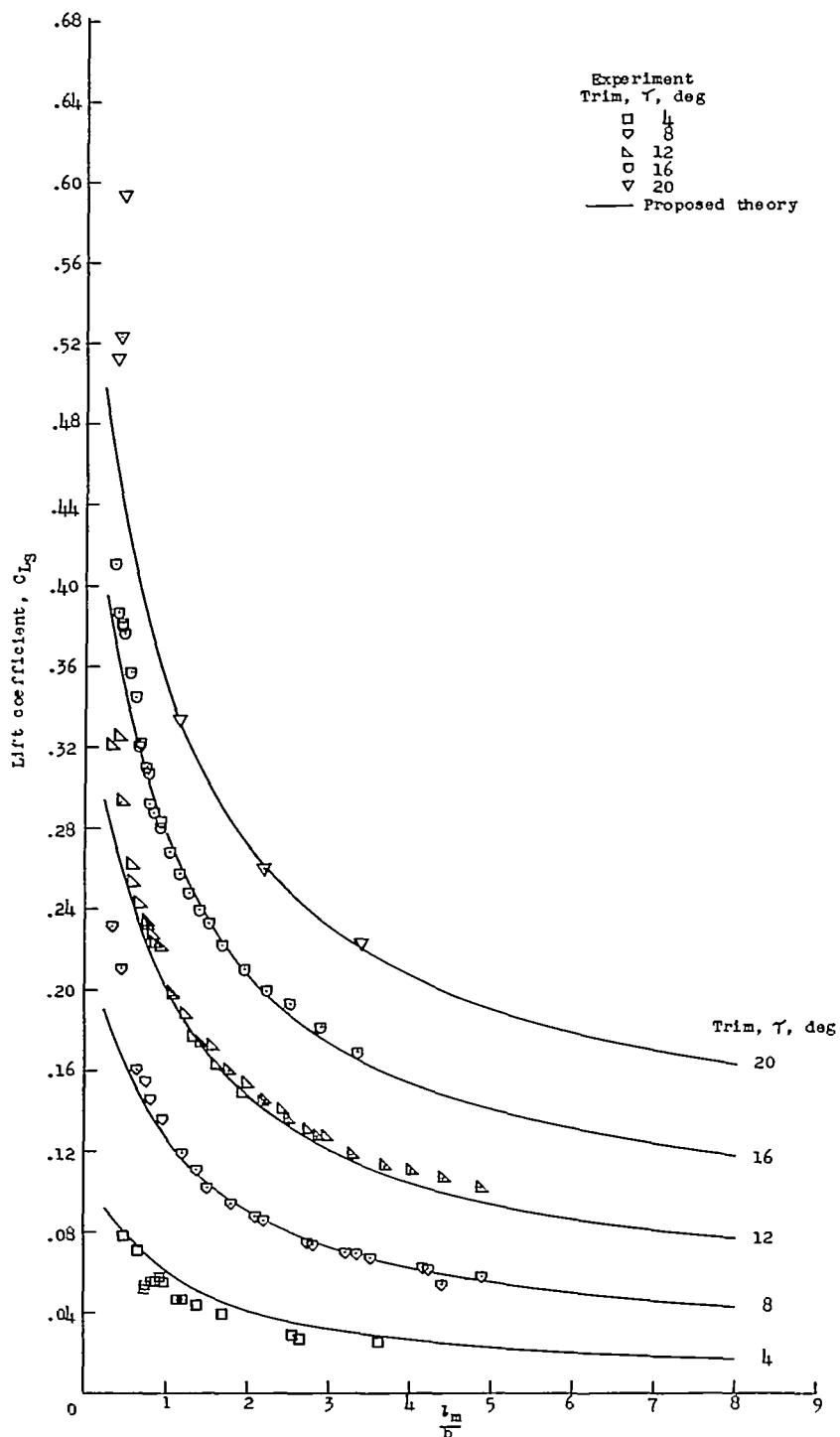
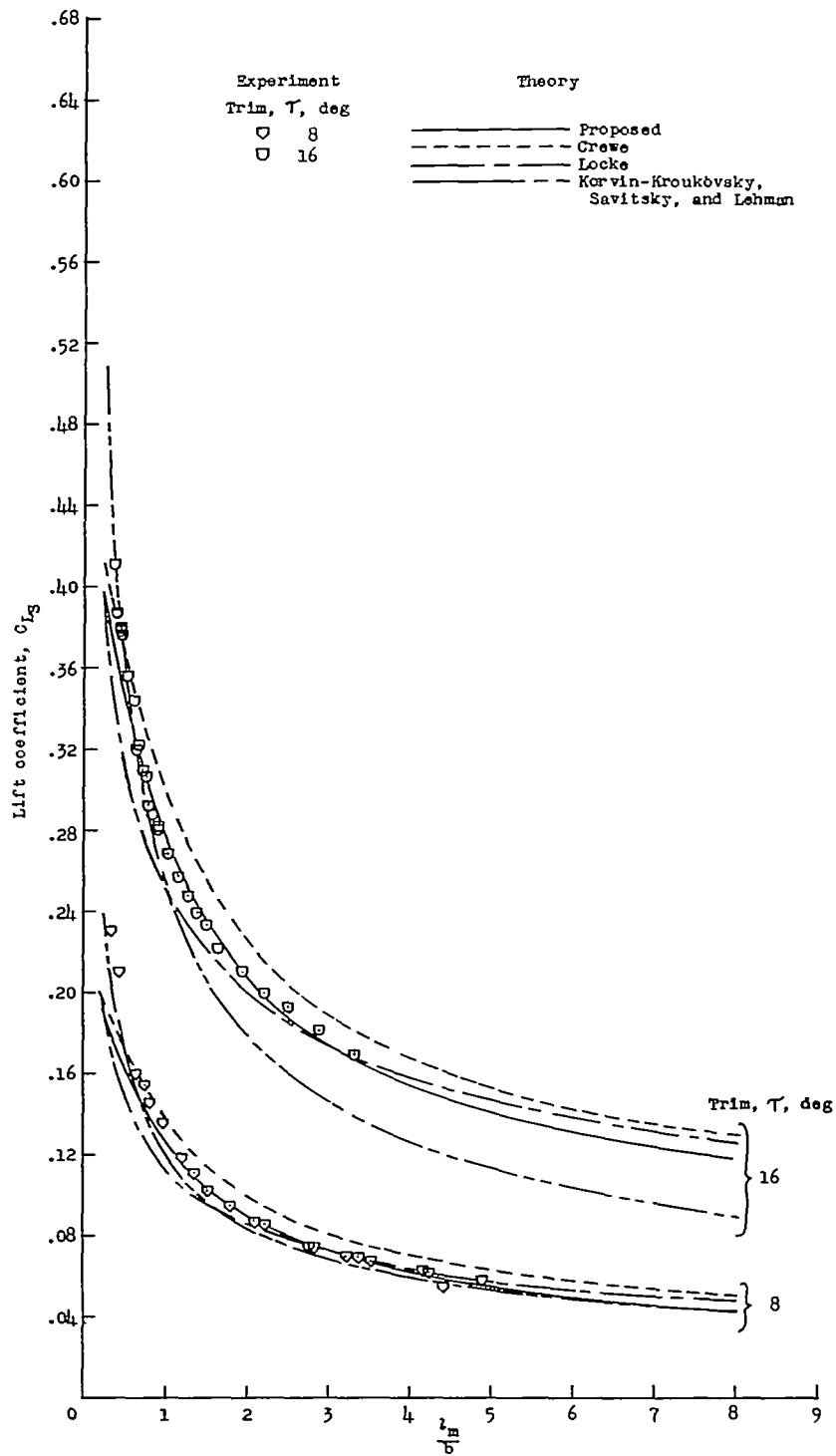Figure 4.- Comparison of proposed theory with experimental lift coeffi-
cients for a 2.5-inch-beam rectangular-flat-plate planing surface
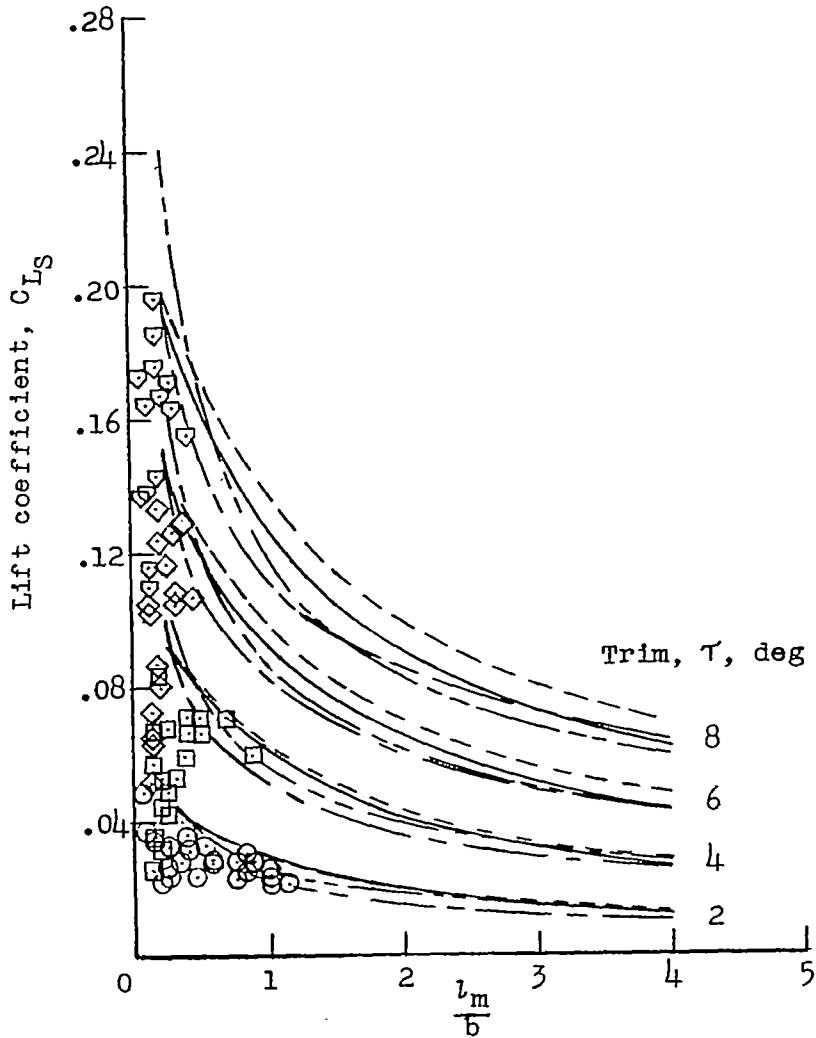(unpublished NACA data).
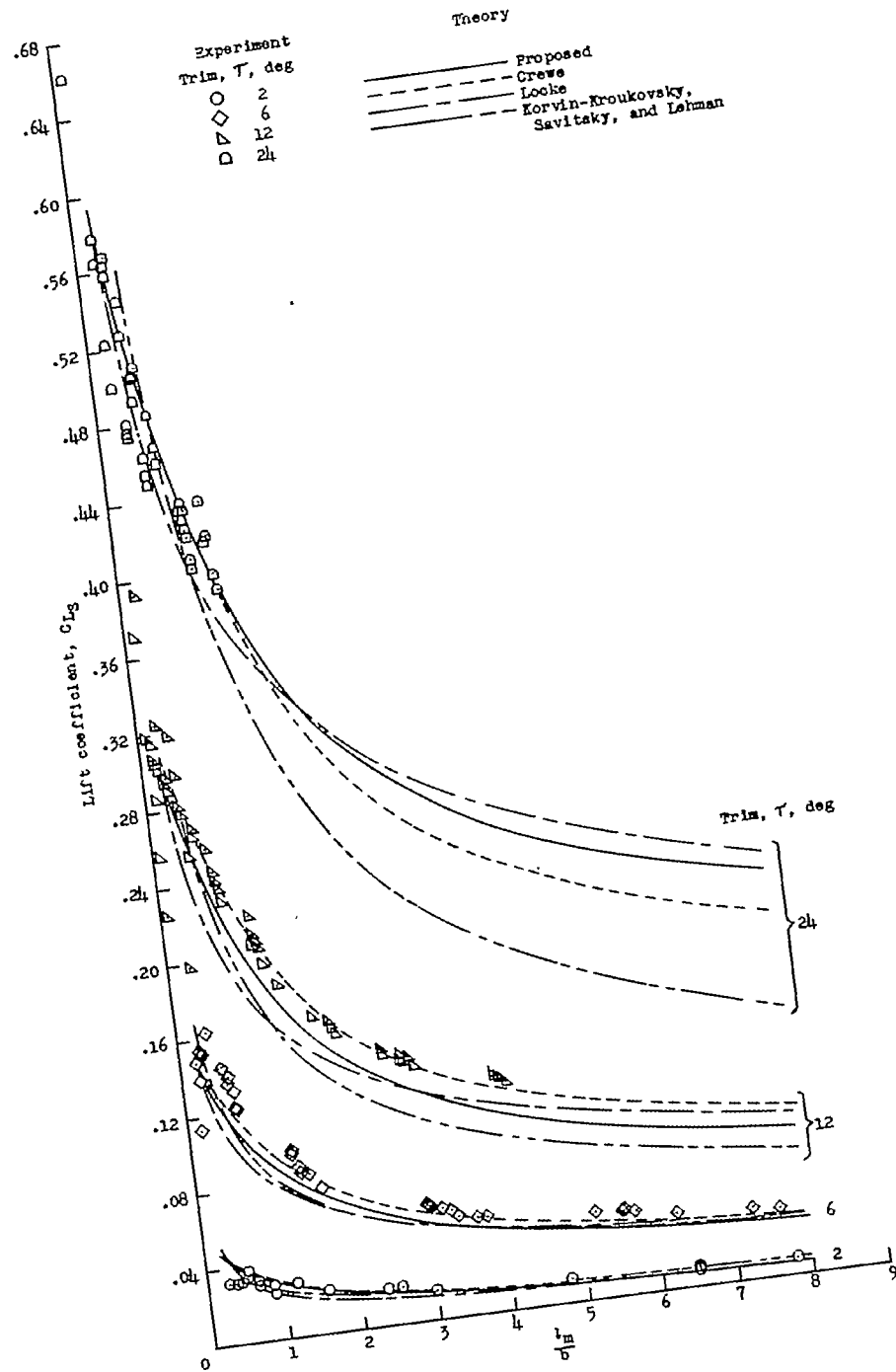
(a) Trim of 2°, 6°, 12°, and 24°.

Figure 5.- Comparison of proposed theory, formulas given by equations (8), (10), and (12), and experimental lift coefficients for the 4-inch-beam rectangular-flat-plate planing-surface data of Weinstein and Kapryan (ref. 25).

26
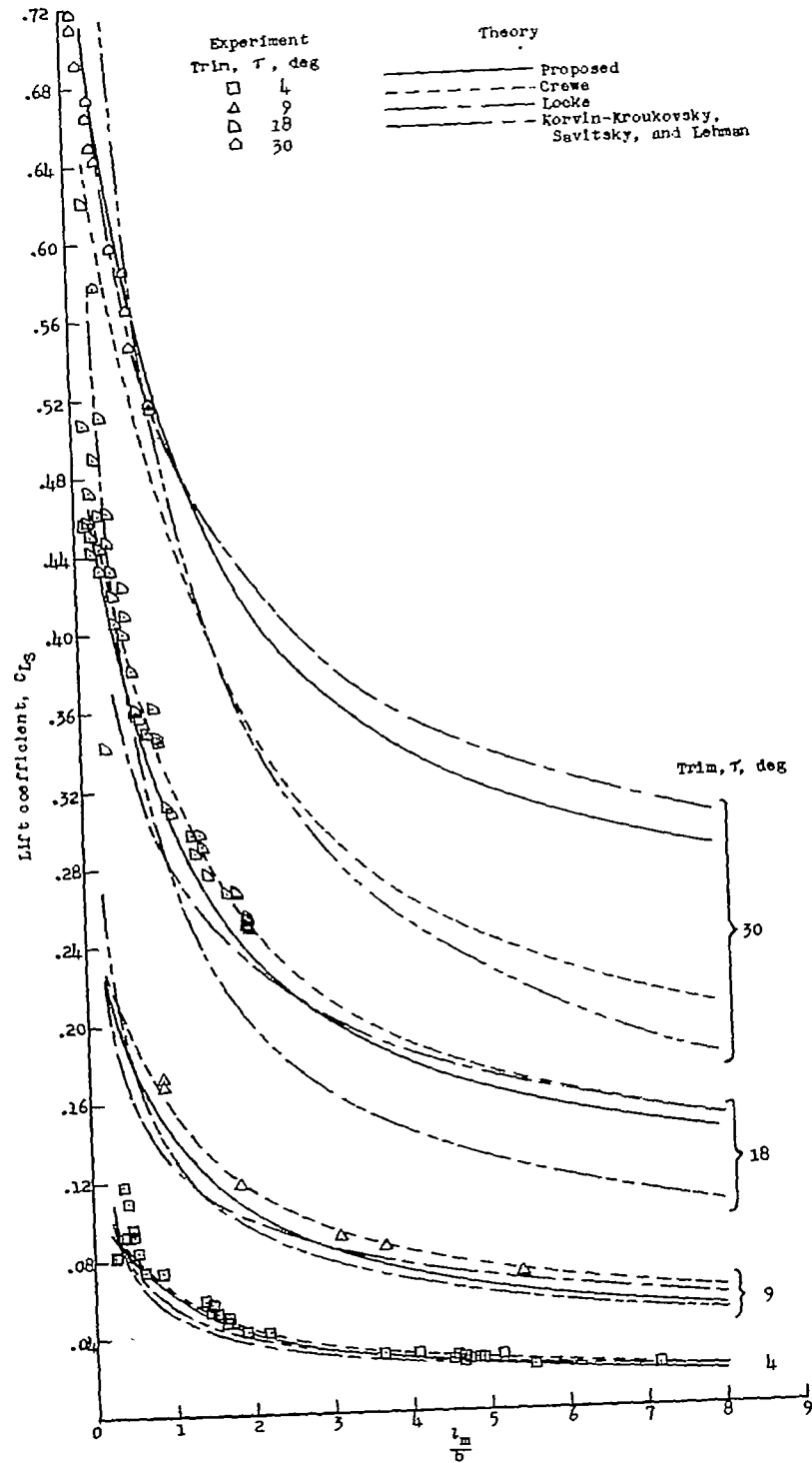
(b) Trim of 4°, 9°, 18°, and 30°.

Figure 5.- Concluded.
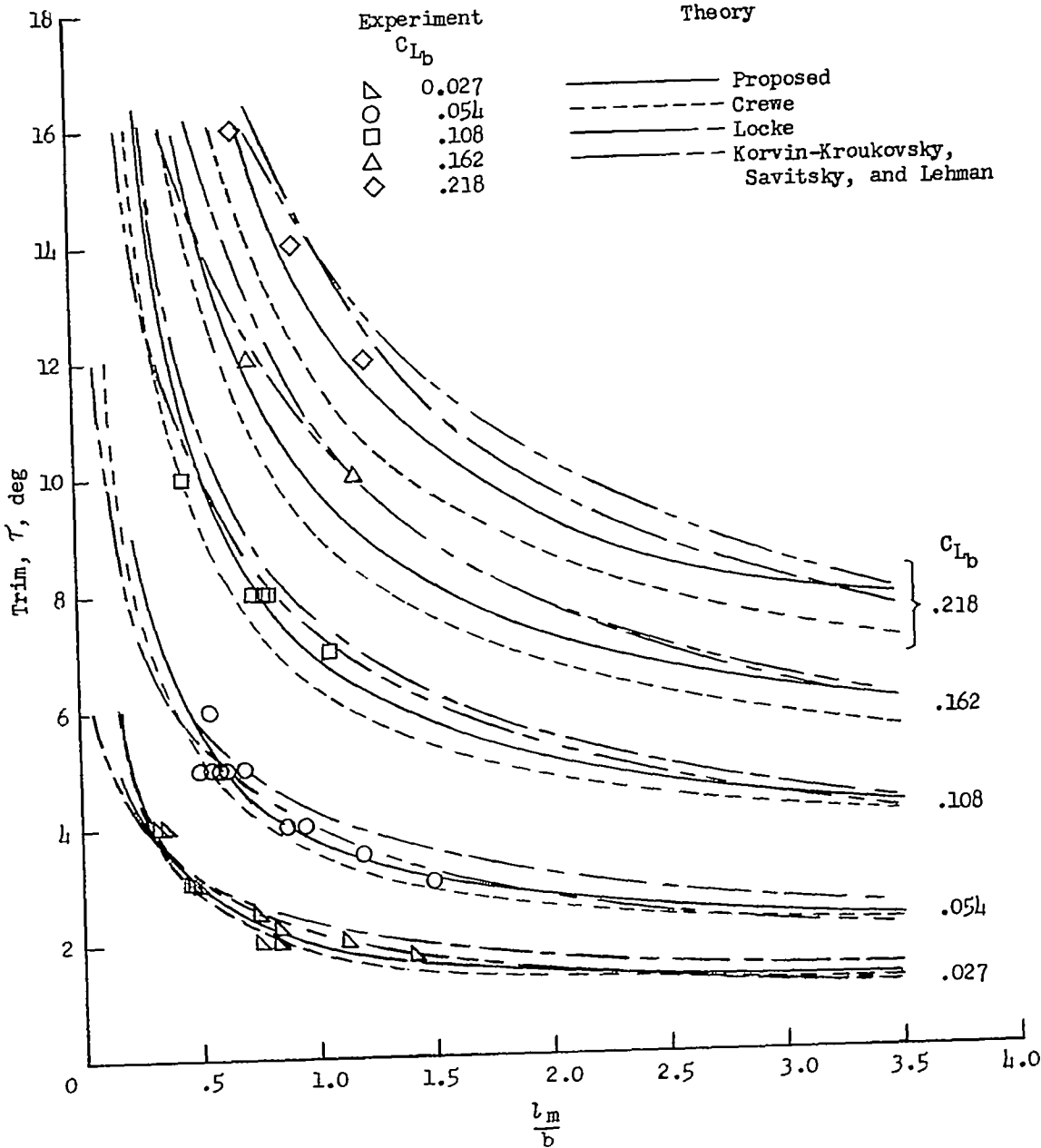
NACA TN 3233
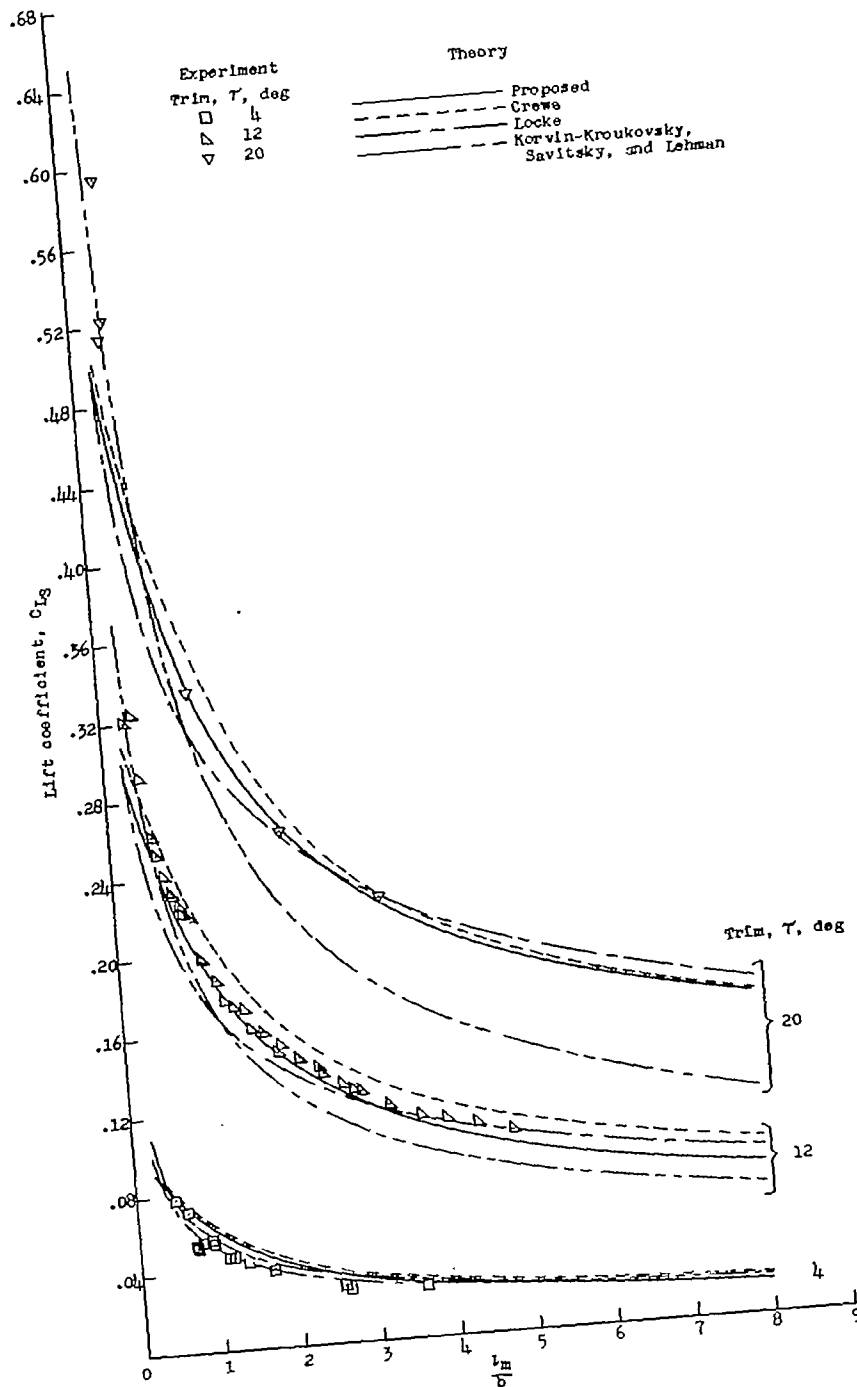
(a) Trim of 4°, 12°, and 20°.

Figure 6.- Comparison of proposed theory, formulas given by equations (8), (10), and (12), and experimental lift coefficients for a 2.5-inch-beam rectangular-flat-plate planing surface (unpublished NACA data).

(b) Trim of 8° and 16°.

Figure 6.- Concluded.

Experiment   Theory
Trim, $\mathcal{T}$, deg

O	2	———————————	Proposed
□	4	— — — — — —	Crewe
◇	6	———— — ————	Locke
▽	8	——————— — —	Korvin-Kroukovsky, Savitsky, and Lehman

Figure 7.- Comparison of proposed theory, formulas given by equations (8), (10), and (12), and experimental lift coefficients for the 16-inch-beam rectangular-flat-plate planing-surface data of Shoemaker (ref. 26).

Figure 8.- Comparison of proposed theory, formulas given by equations (8), (10), and (12), and experimental lift coefficients for the 4-inch-beam rectangular-flat-plate planing-surface data of Locke (ref. 27).
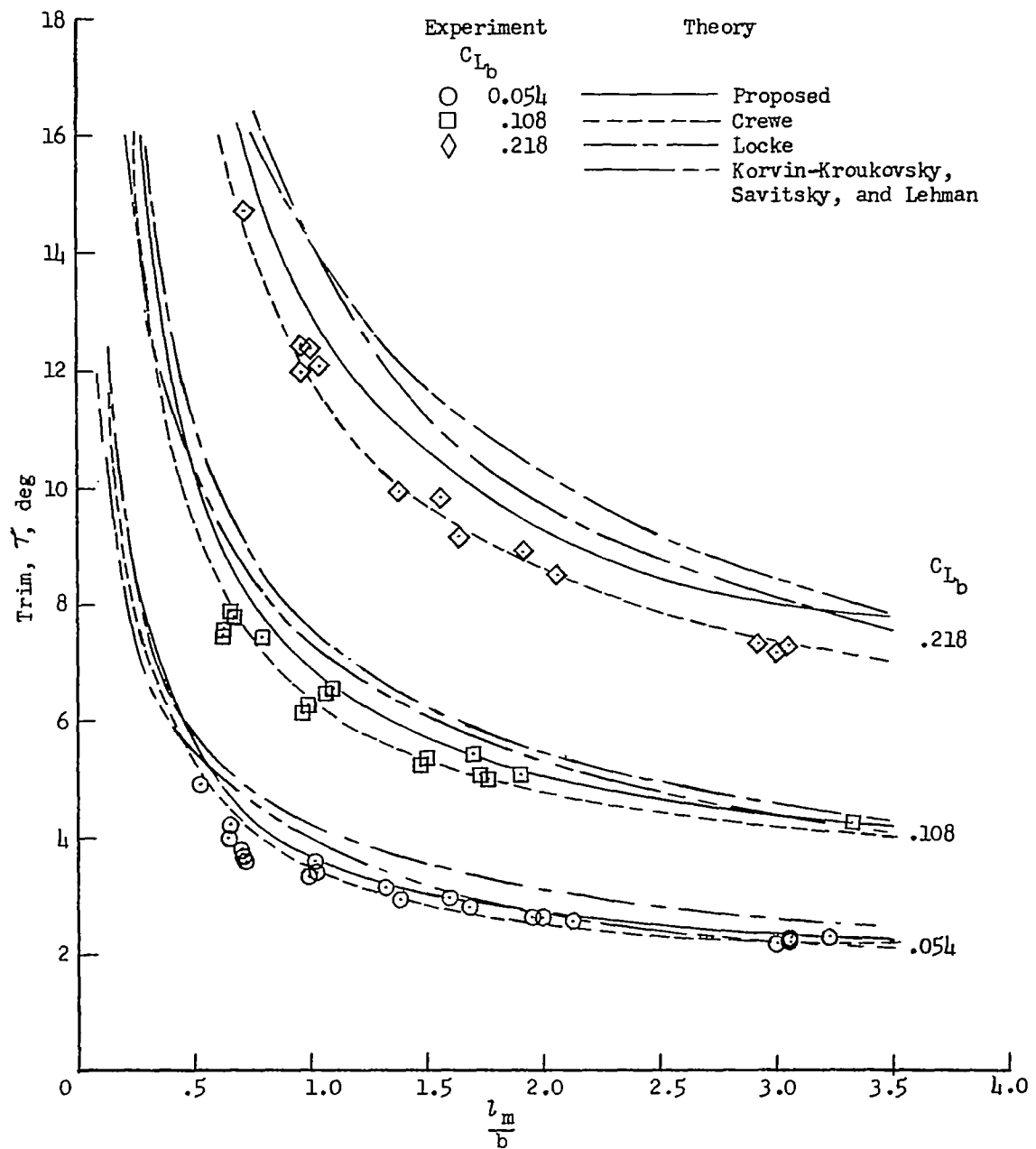
Figure 9.- Comparison of proposed theory, formulas given by equations (8), (10), and (12), and experimental lift coefficients for the 15- and 30-centimeter-beam (5.91- and 11.81-inch-beam) rectangular-planing-surface data of Sambraus (ref. 16).
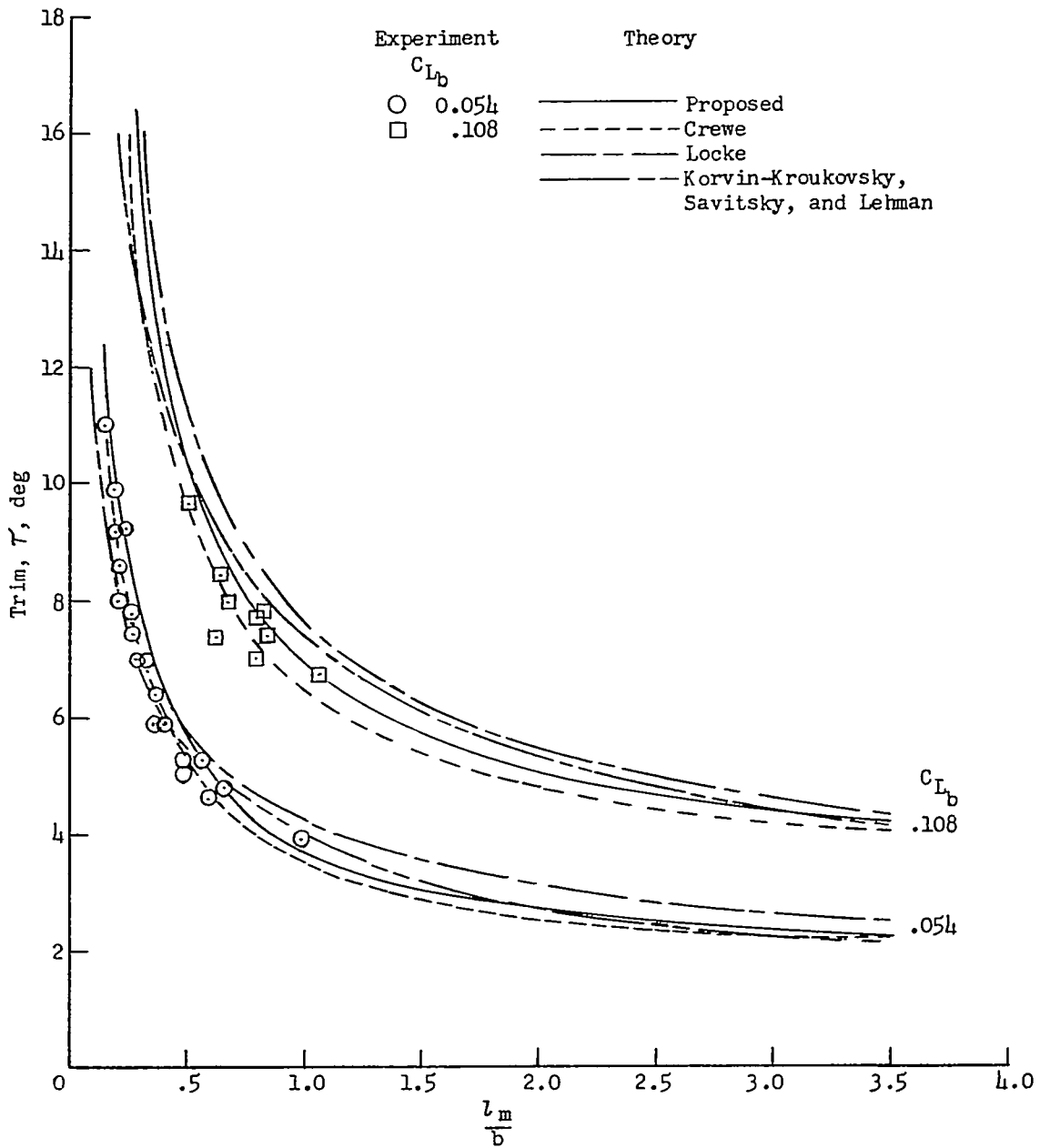
Figure 10.- Comparison of proposed theory, formulas given by equations (8),
(10), and (12), and experimental lift coefficients for the 30-centimeter-
beam (11.81-inch-beam) rectangular-planing-surface data of Sottorf
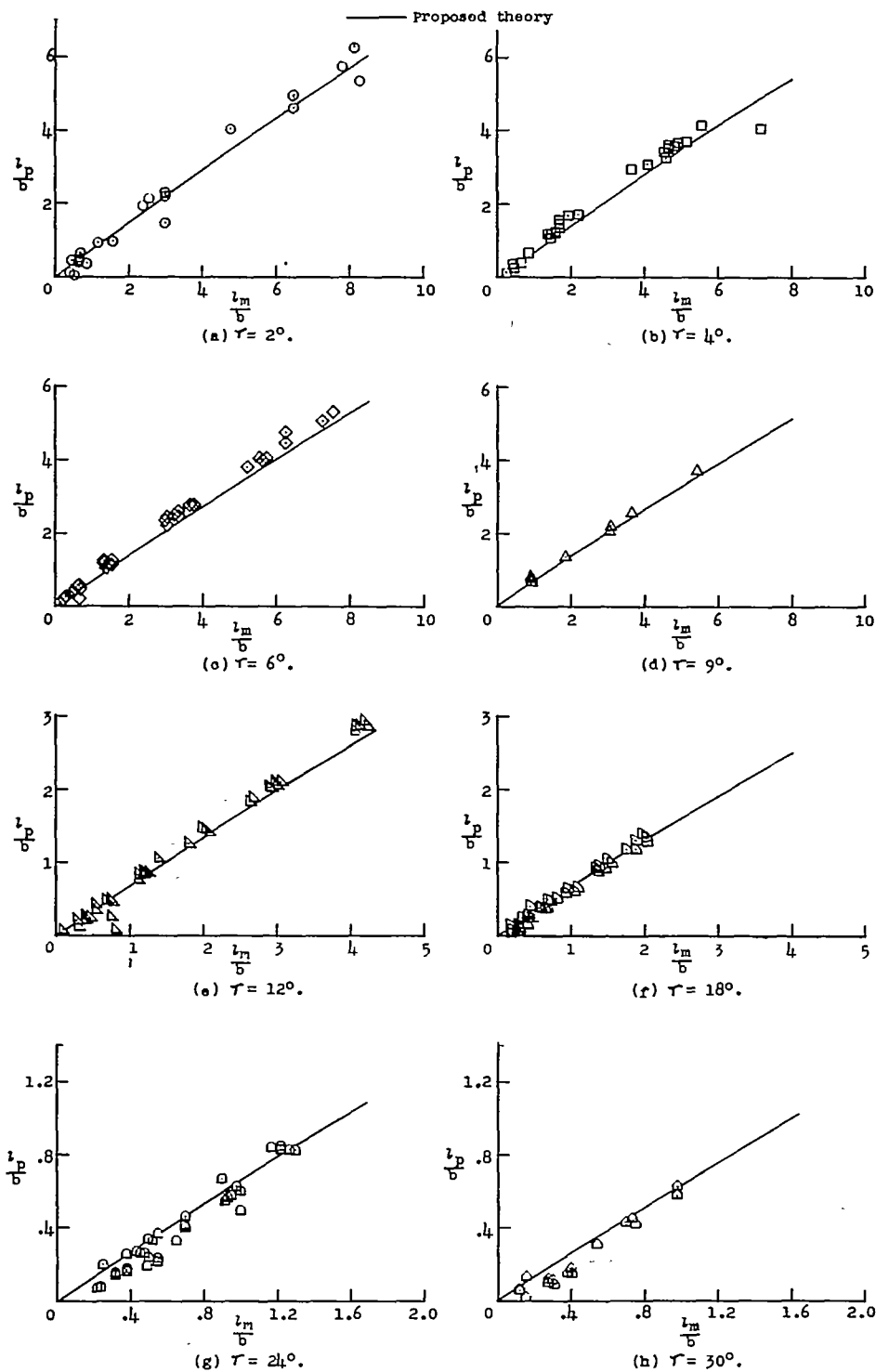(ref. 15).

Figure 11.- Variation of center-of-pressure ratio with mean wetted-length—
beam ratio for the 4-inch-beam rectangular-flat-plate planing-surface
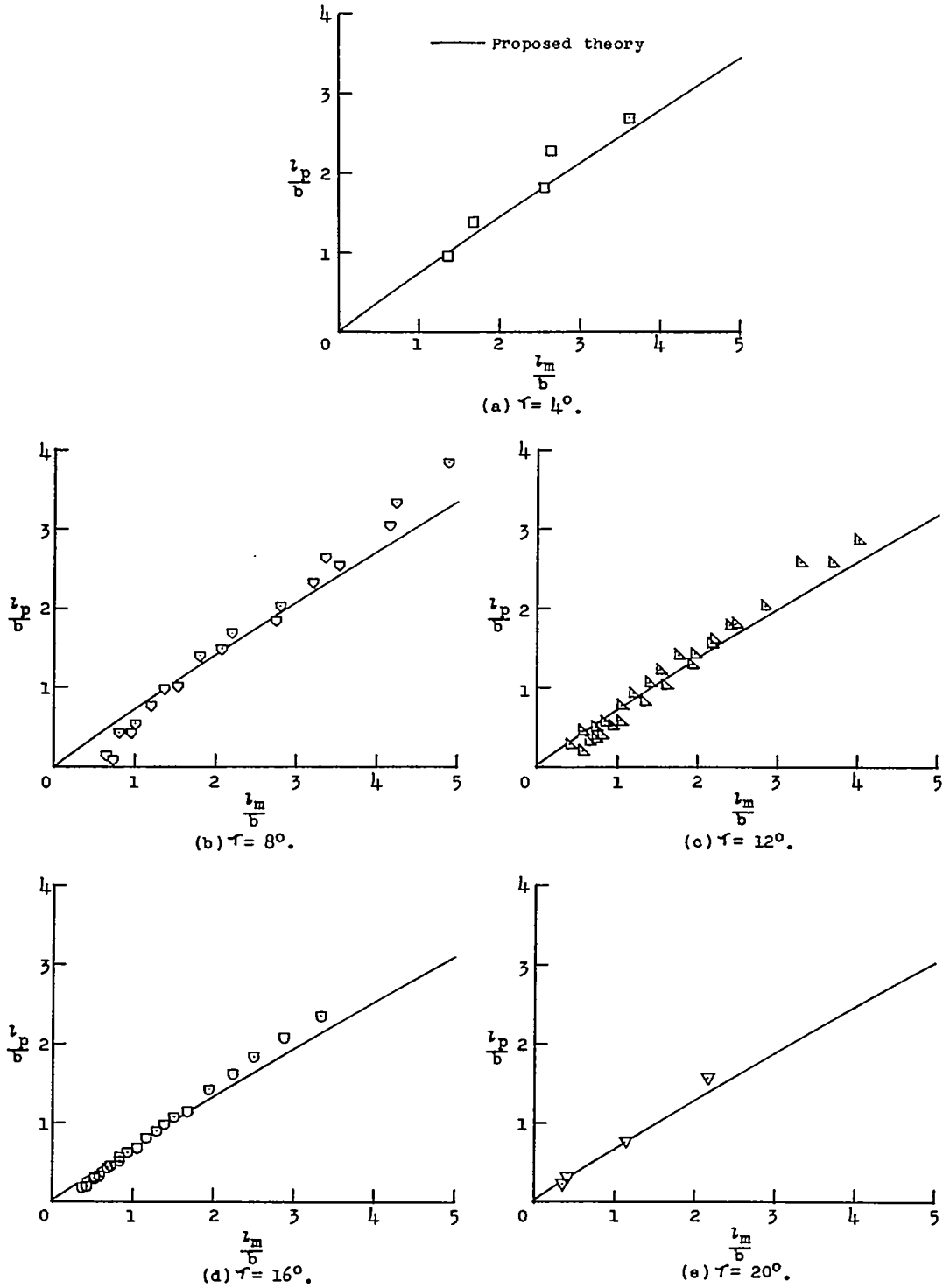data of Weinstein and Kapryan (ref. 25).

Figure 12.- Variation of center-of-pressure ratio with mean wetted-length—
beam ratio for a 2.5-inch-beam rectangular-flat-plate planing surface
(unpublished NACA data).